

# Development of Multiple Lyapunov Functions for a Hybrid Power System with a Tap Changer

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**Abstract:** This term paper looks into multiple Lyapunov functions as applied to power system examples. Two example power systems are considered, one with constant real load and variable reactive load (one dimensional system); and other with both variable real and reactive loads (two dimensional system). Lyapunov functions are constructed and their validity is confirmed by simulations. It is observed that the stable switched system obeys the stability criterion in terms of multiple Lyapunov functions [1].

## 1. Introduction

Power systems are in general described by coupled nonlinear differential-algebraic equations (DAE). A control device such as a tap changing transformer introduces discrete events in the system operation as the system description changes abruptly when the tap changer changes its secondary turns ratio from one value to another in a vanishingly small time. A function of a tap changer is to boost or lower a voltage of the secondary bus (node) by undergoing such a discrete change of parameter. This switching event when superimposed over the continuous dynamics of generators and loads, leads to a hybrid behavior. Theoretical analysis of a hybrid power system becomes difficult due to nonlinear differential equations and implicit nonlinear algebraic equations, which are coupled together.

Figure 1 shows a power system under consideration. System data is taken from [2]. The generator end and the consumer load end are connected by a transmission line with a tap changing transformer in series. The load side is described by a single differential equation making the whole system one-dimensional. This dynamical equation corresponds to voltage dependent reactive power. There are no dynamics considered for generators for simplicity.

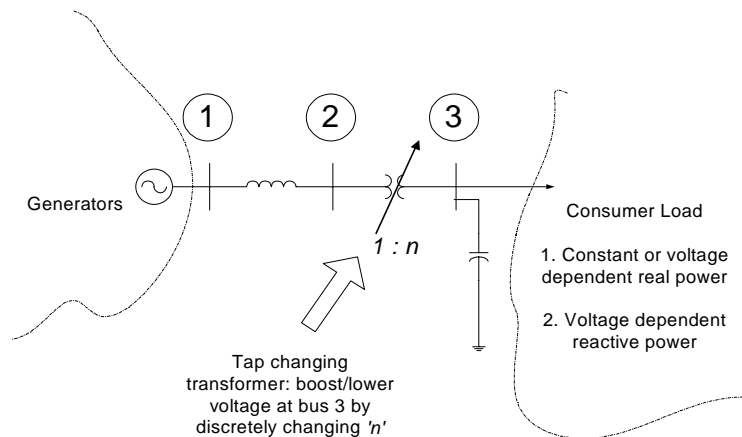


Figure 1: power system with a tap-changing transformer

Power systems are, in general, not globally stable. After an occurrence of a disturbance such as tripping of a transmission line or switching in of a load, the operating point moves to a different location in state-space. Efforts are taken in such a way that the initial point for the post-disturbance dynamics lies in the region of attraction for stability. Apart from stability, the system needs to be brought back to a good operating point for quality service (e.g. to maintain voltages at rated values). This is done in various ways, one of which is by using a tap changing transformer. Though a discrete tap changing in a right manner is beneficial from stability and quality point of view, it may introduce unwanted limit cycles in the system.

In this paper, we find multiple Lyapunov functions for the system in figure 1 and monitor its behavior under switching events.

## 2. Schematic diagram from a control systems point of view

Figure 2 shows a schematic diagram for a tap changing process. The plant is DAE with 'n' as a discrete parameter. For the system at hand, the plant description is given by (1).

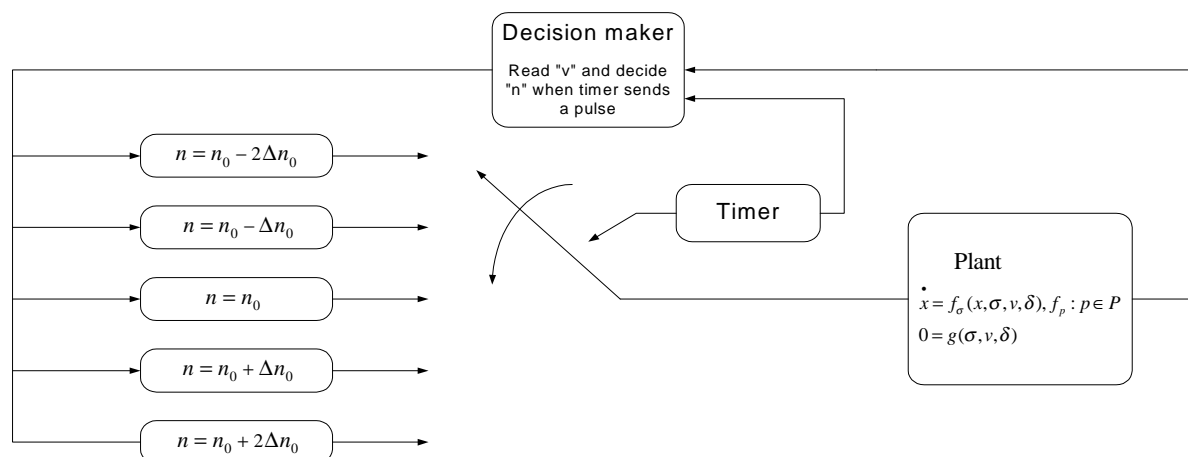


Figure 2: Schematic of figure 1

Plant description

$$\dot{x} = \frac{1}{T}(Q_s - x - Q_s v)$$

$$0 = P_s + \left(\frac{v_0}{nx_1}\right)v \sin \delta$$

$$0 = \left(\frac{v_0}{nx_1}\right)(nv_0 - v \cos \delta) - x - \left(Q_s - \frac{1}{x_2}\right)v^2$$

Variables are:

$x$ : continuous

$n$ : discrete

$v, \delta$ : algebraic

(1)

The plant output is the load voltage 'v' that is fed to a decision maker which makes a decision for 'n' when it receives a switching pulse from the timer. The timer represents an inherent time delay in the tap changer control system. It should be noted that despite of being a scalar system, nonlinearity makes the system quite complex.

### 3. Finding a Lyapunov function for a fixed ‘n’

As a power system is not globally stable and has unstable equilibrium points, the Lyapunov function may not be defined over the entire state-space. Hence, we can find a function that is strictly increasing and positive definite only over a region of interest. Let us define a time derivative of the candidate function as (2). By integrating (2), we have the unnormalized Lyapunov function as in (3). Refer Appendix A for details.

$$\dot{V} = -\frac{2}{T}(Q_s - x - Q_s v^2)^2, T > 0 \quad (2)$$

$$V = x^2 + \frac{k_1}{n} \left(1 - \frac{2}{3} k_2 n^2 - \frac{v^2}{3}\right) \sqrt{v^2 - k_2 n^2} + k_3 v^2 \left(1 - \frac{v^2}{2}\right) \quad (3)$$

$$\text{where } k_1 = \frac{2Q_s v_0}{x_1 n}, k_2 = \frac{n P_s x_1}{v_0}, k_3 = 2Q_s \left(Q_s - \frac{1}{x_2}\right)$$

The Lyapunov function may be normalized by defining it w.r.t. the stable equilibrium point (s.e.p.) as in (4).

$$V_{\text{normalized}} = V - V_{\text{s.e.p.}} \quad (4)$$

Figure 3 (a) and 3 (b) show the plots of the unnormalized and normalized function respectively. It is observed from 3 (a) that it is strictly increasing and positive definite for  $n=0.7$ , but it loses these properties as  $n$  is further increased. We concentrate only on the region where  $-5 \leq x \leq 5$  which happens to be the region of interest. It can be indeed verified from simulations that the system moves towards the stability margin as  $n$  is increased from 0.7 and loses stability at around  $n=1.2$ . Thus, there is no Lyapunov function existing beyond  $n=1.2$ . Figures 3 (a, b) provide precisely the same information. The bottom of the valley corresponds to the stable equilibrium point which vanishes when  $n \approx 1.2$ . We can work with this candidate function when  $n < 1.2$  and  $-5 \leq x \leq 5$ . The normalized function in 3 (b) is zero at the s.e.p. (bottom of the valley).

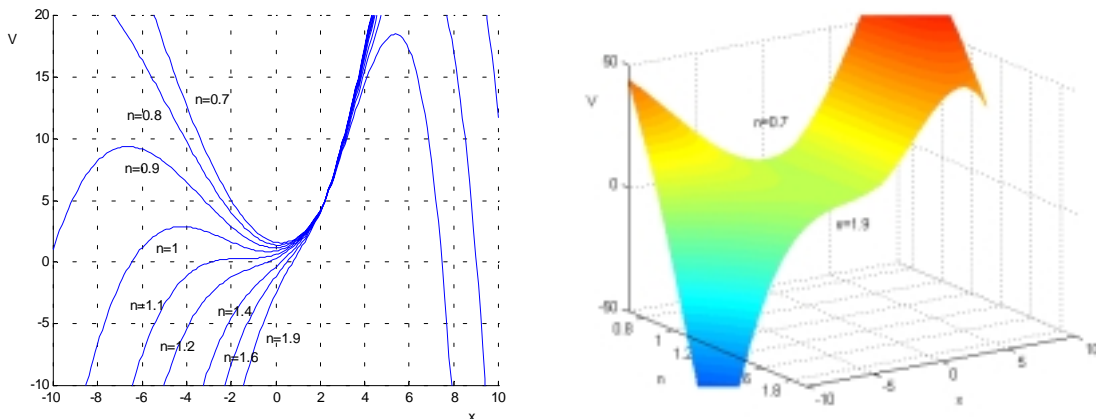


Figure 3 (a): Unnormalized Lyapunov function

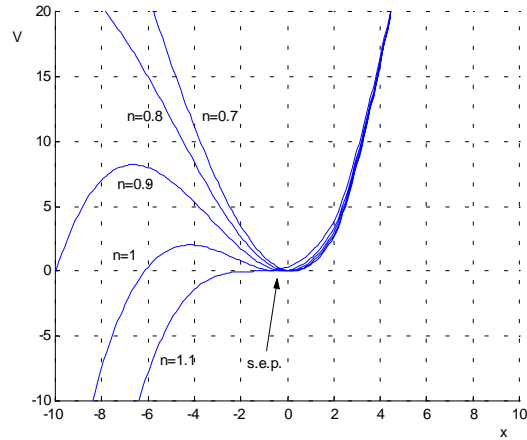


Figure 3 (b): Normalized Lyapunov function

#### 4. Simulation of the hybrid system and multiple Lyapunov functions

As one can see, the definition of the Lyapunov function changes as  $n$  is varied, thus we have multiple Lyapunov functions to work with, each valid for corresponding  $n$ . Work on multiple Lyapunov functions is contained in [1], [3-5]. As evident from (2) and (3), the Lyapunov function is decreasing with time whenever positive definite, hence every subsystem that makes it positive definite  $\forall t$ , is stable.

The timer sends a pulse to the decision maker periodically. Decision maker compares the load voltage with a reference voltage. The taps ( $n$ ) are increased or decreased by one tap step depending upon whether the load voltage is lower or higher than the reference voltage. This is the instant when switching occurs. A deadband is included to avoid chattering.

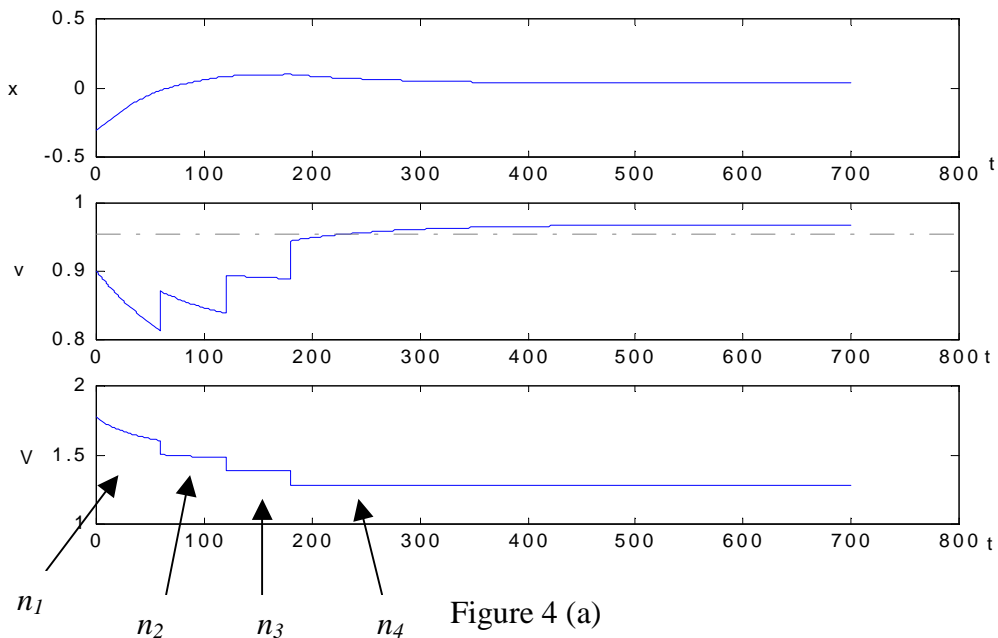
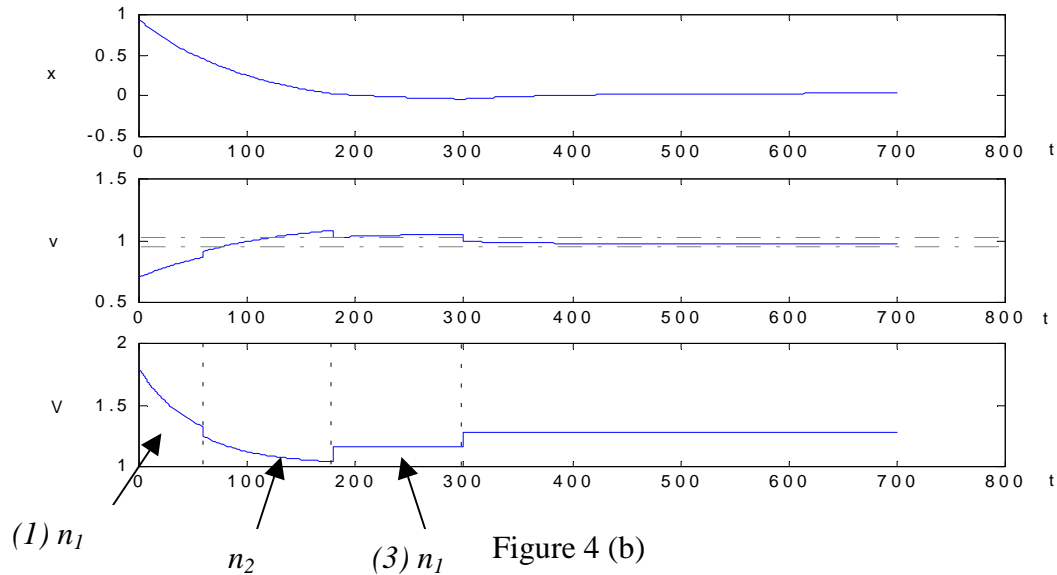


Figure 4 (a)

Figures 4 (a-d) show time domain simulation of this system with abovementioned switching logic, each with different initial conditions. The deadband region is shown with a horizontal dashed line. In every figure, the first trajectory is of the continuous variable  $x$ , second is the algebraic variable  $v$  and the third is the unnormalized Lyapunov function  $V$ . The regions between switching events are shown by corresponding values of  $n$ .

As expected,  $v$  and the Lyapunov function  $V$  have discontinuities at the instants of switching (refer figure 4 (a)).  $V$  decreases as time progresses. Since individual subsystems for this simulation are stable, the hybrid system is stable too.



For the simulation in figure 4 (b), we can apply the stability criterion for hybrid systems. First and third subsystems correspond to the same subsystem ( $n_1$ ) and the value of  $V$  at the start of subsystem 3 ( $V_2$ ) is less than that for subsystem 1 ( $V_1$ ). By the theorem of stability of switched system, the system is stable [1], i.e.  $V_1 > V_2$ . It is observed that no more switching occurs when  $v$  settles down within the deadband.

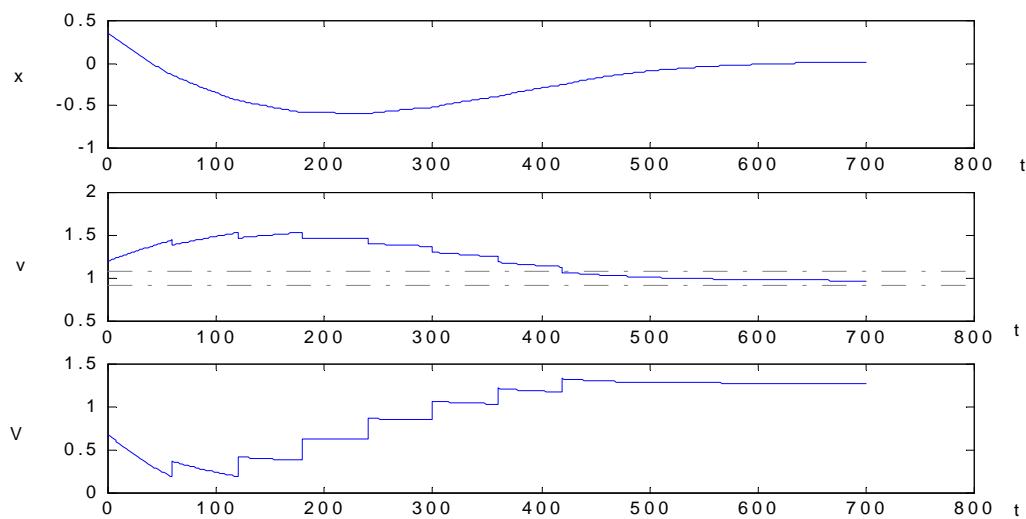


Figure 4 (c)

The Lyapunov function is increasing with time in this case (figure 4 (c)) as it is unnormalized. It does not go to zero in the steady state. But the system is stable as all subsystems are stable and switching occurs between different subsystems.

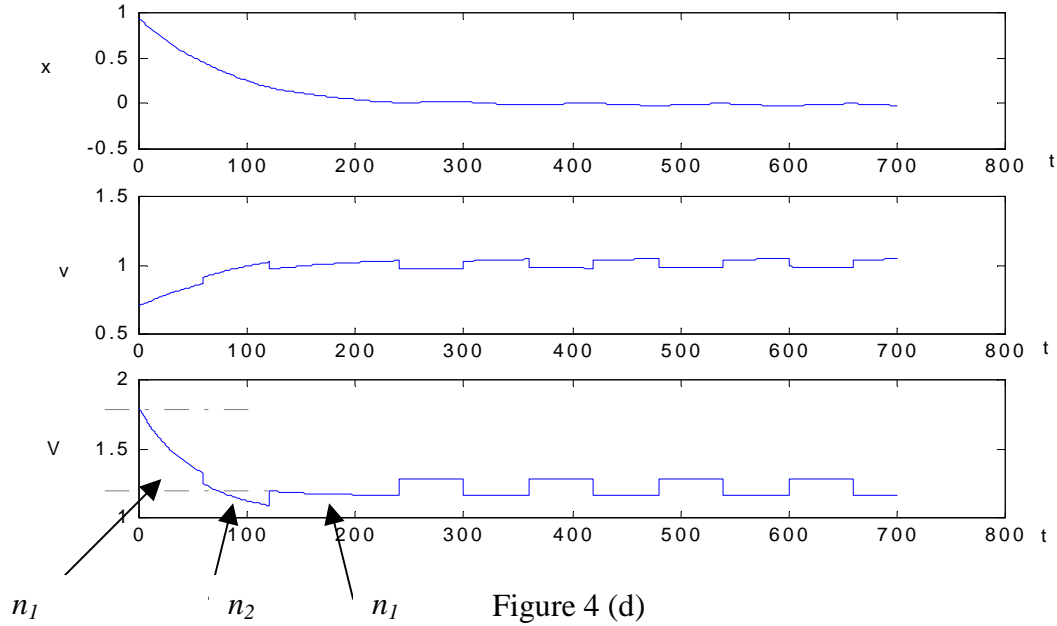


Figure 4 (d)

Stability criterion in terms of Lyapunov function can be again applied for regions 1 and 3 in figure 4 (d) to conclude that the system is stable. The unnormalized Lyapunov function oscillates in steady state when the system enters the limit cycle. This is chattering and is introduced by insufficient deadband.

The Lyapunov function used in all these simulations is unnormalized. One may use a normalized function with possibly no better observations.

## 5. Two dimensional system

A more realistic, two-dimensional system is now considered where we have one additional differential equation. The system still looks as in figure 1 but now the load has voltage dependent (instead of constant) real power. The continuous dynamic state variables are denoted as  $x_p$  and  $x_q$ . Mathematical description of the system is given by (5). The candidate Lyapunov function is defined as in (6) and (7).

$$\begin{aligned} \dot{x}_p &= \frac{1}{T_p} (P_s - x_p - P_s v^2) \\ \dot{x}_q &= \frac{1}{T_q} (Q_s - x_q - Q_s v^2) \end{aligned} \tag{5}$$

$$0 = x_p + P_s v^2 + \frac{v_0}{n x_1} v \sin \delta$$

$$0 = x_q + (Q_s - \frac{1}{x_2}) v^2 - \frac{v_0}{n x_1} (n v_0 - v \cos \delta)$$

$$\begin{aligned}\dot{V} &= -\frac{2}{T_p}(P_s - x_p - P_s v^2)^2 - \frac{2}{T_q}(Q_s - x_q - Q_s v^2)^2; T_p, T_q > 0 \\ &= -2T_p(\dot{x}_p)^2 - 2T_q(\dot{x}_q)^2 < 0\end{aligned}\tag{6}$$

$$\begin{aligned}V &= -2T_p \int_{x_p^e}^t (\dot{x}_p)^2 dt - 2T_q \int_{x_q^e}^t (\dot{x}_q)^2 dt \\ &= \{x_p^2 - (x_p^e)^2\} + \{x_q^2 - (x_q^e)^2\} - 2\{P_s \int_{x_p^e}^{x_p} (1-v^2) dx_p + P_q \int_{x_q^e}^{x_q} (1-v^2) dx_q\}\end{aligned}\tag{7}$$

Equation (7) shows an expression for a normalized Lyapunov function. Refer appendix B for further details.

Figure 5 shows simulation of the continuous system (5) without any switching events. The normalized Lyapunov function  $V$  as defined in (7) goes to zero in steady state.

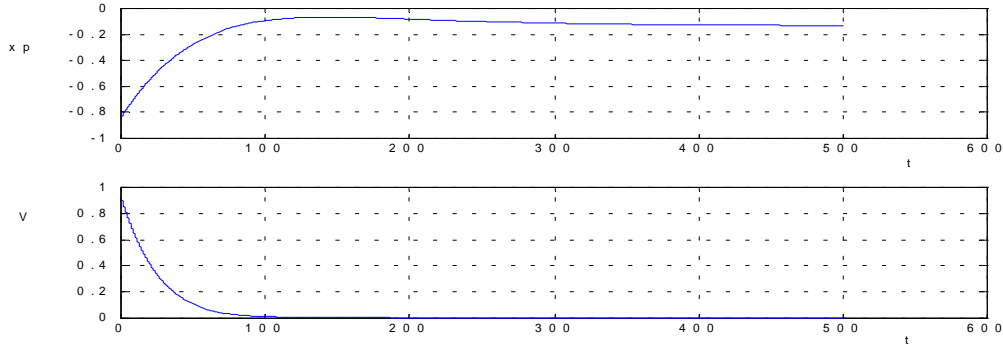


Figure 5: Continuous dynamical system (5) and its normalized Lyapunov function (7)

Using the same switching logic as in section 4, simulations are done for (5). Results for different initial conditions are summarized in figure 6.

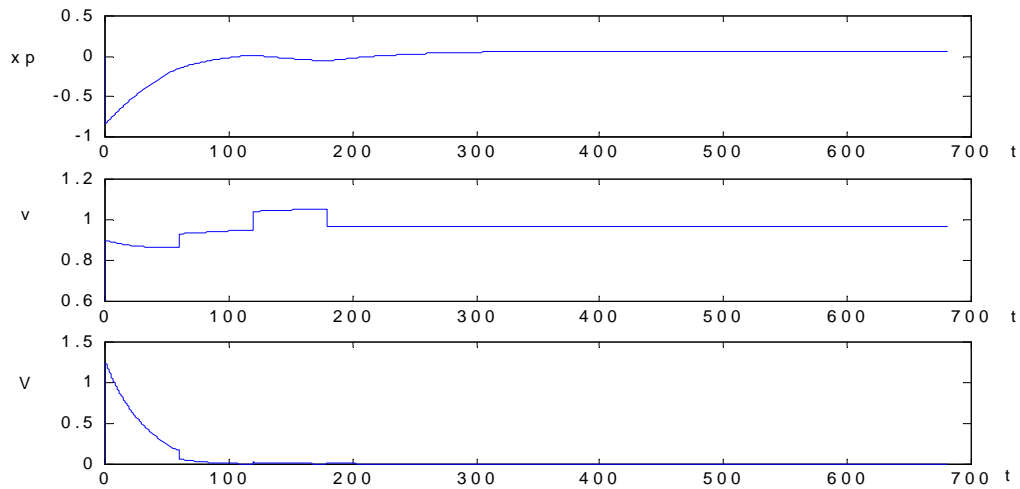


Figure 6 (a)

Though the Lyapunov function is barely visible after  $t=100$  s, it decreases to zero in the steady state. The switched system is stable.

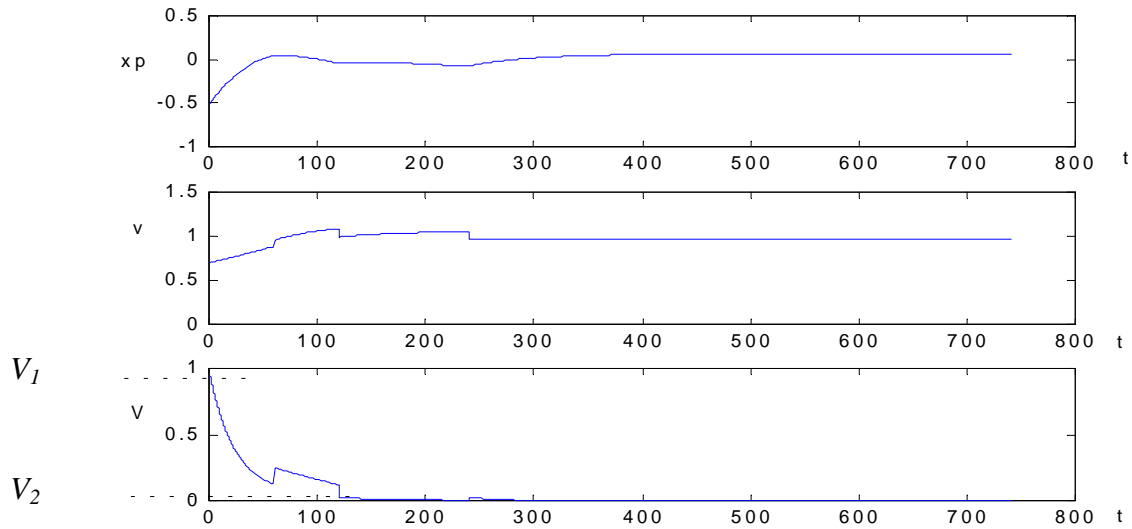


Figure 6 (b)

As evident from figure 6 (b),  $V$  at the starting of region 3 is lower than at the starting of region 1 i.e.  $V_1 > V_2$ . Hence, the switched system is stable.

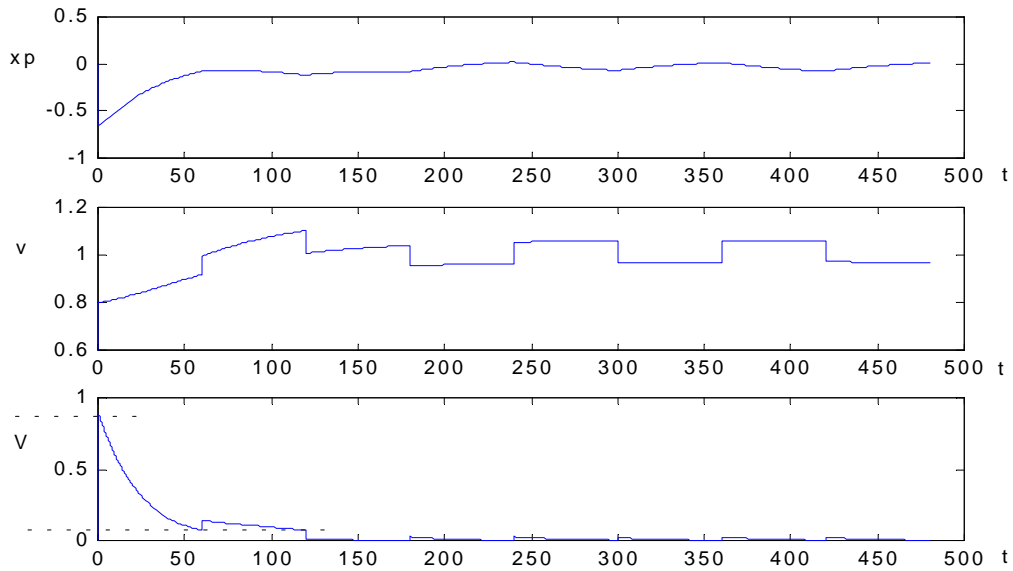


Figure 6 (c)

The system jumps into a limit cycle (chattering) in the steady state in figure 6 (c) due to inadequate deadband.  $V$  also oscillates in the steady state. Yet, the system is stable (not asymptotically stable).

The function is positive definite and decreasing along a continuous trajectory between events. Thus, this candidate function looks a valid Lyapunov function in the region of interest.

## 6. Conclusions

In this term project, multiple Lyapunov functions are defined for a small power system and the criterion for a stable hybrid behavior is verified by simulations. I could not come up with a case where a fast switching between two stable subsystems produces an unstable hybrid system. It is possible that this kind of behavior is not possible with the simple system that I considered. Though simple, the systems considered become quite difficult for analysis and computations owing to the nonlinearity of the DAE and implicitness of the algebraic equations in power systems.

## 7. References

- [1] Daniel Liberzon, A. Stephan Morse, "Basic Problems in Stability and Design of Switched Systems", *IEEE Control Systems Magazine*, to appear.
- [2] Q. Wu, D. H. Popovic, D. J. Hill, "Avoiding Sustained Oscillations in Power Systems with Tap Changing Transformers", *Electrical Power and Energy Systems*, vol 22, 2000, pp 597-605.
- [3] M. S. Branicky, "Multiple lyapunov functions and other analysis tools for switched and hybrid systems", *IEEE transactions on automatic control*, Vol. 43, No. 4, pp. 475-482, April 1998.
- [4] M. Johansson, A. Rantzer, "Computation of piecewise lyapunov functions for hybrid systems", *IEEE transactions on automatic control*, Vol. 43, No. 4, pp. 555-559, April 1998.
- [5] M. A. Wicks, P. Peleties, "Construction of piecewise lyapunov functions for stabilizing switched systems", *Proceedings of the 33<sup>rd</sup> conference on Decision and Control*, Lake Buena Vista, FL, pp-3492-3497, Dec 1994.

## 8. Appendices

Appendix A: Derivation of  $V$  from  $\dot{V}$  for the one-dimensional hybrid system example

$$\begin{aligned}\dot{V} &= -\frac{2}{T}(Q_s - x - Q_s v^2)^2 \\ &= -\frac{2x}{T}(Q_s - x - Q_s v^2) - \frac{2Q_s}{T}(Q_s - x - Q_s v^2)(1 - v^2) \\ &= 2x\dot{x} - \frac{2Q_s}{T}(Q_s - x - Q_s v^2)(1 - v^2)\end{aligned}\quad (1A)$$

From (1),

$$\begin{aligned}\dot{v} &= -\frac{(Q_s - x - Q_s v^2)}{T\left(\frac{v_0}{x_1 n \cos \delta} + 2\left(Q_s - \frac{1}{x_2}\right)v\right)} \\ \dot{\delta} &= \frac{P_s n x_1}{v_0 \cos \delta} \dot{v}\end{aligned}\quad (2A)$$

Substituting (2A) into (1A),

$$\dot{V} = 2x\dot{x} + 2Q_s v_0 (1 - v^2) \frac{\dot{v}}{x_1 n \cos \delta} + 4Q_s \left(Q_s - \frac{1}{x_2}\right) v (1 - v^2) \dot{v}\quad (3A)$$



$$M = \frac{2Q_s v_0 v (1 - v^2) \dot{v}}{x_1 n \sqrt{v^2 - \left(\frac{n P_s x_1}{v_0}\right)^2}}$$

With a change of variable  $\theta^2 = v^2 - \left(\frac{n P_s x_1}{v_0}\right)^2$

$$M = \frac{2Q_s v_0 \left\{1 - \theta^2 - \left(\frac{n P_s x_1}{v_0}\right)^2\right\}}{x_1 n} \dot{\theta}\quad (4A)$$

Substituting (4A) into (3A) and integrating (3A), one gets (3).

Appendix B: Derivation of  $V$  from  $\dot{V}$  for the two-dimensional hybrid system example

$$\begin{aligned}
\dot{V} &= -\frac{2}{T_p}(P_s - x_p - P_s v^2)^2 - \frac{2}{T_q}(Q_s - x_q - Q_s v^2)^2 \\
&= \frac{2x_p}{T}(P_s - x_p - P_s v^2) - \frac{2P_s}{T_p}(P_s - x_p - P_s v^2)(1 - v^2) \\
&\quad + \frac{2x_q}{T_q}(Q_s - x_q - Q_s v^2) - \frac{2Q_s}{T_q}(Q_s - x_q - Q_s v^2)(1 - v^2) \\
&= 2x_p \dot{x}_p + 2x_q \dot{x}_q - 2P_s(1 - v^2) \dot{x}_p - 2Q_s(1 - v^2) \dot{x}_q
\end{aligned} \tag{1B}$$

Integration of (1B) gives (7). It is difficult to find an explicit expression for  $V$ , hence numerical integration is adopted to evaluate the last term in (7) along the trajectory. While simulating the hybrid system, stable equilibrium point is computed for each subsystem (for a particular value of  $n$ ) to evaluate the Lyapunov function in (7).